MA 587/687

Name (Print last name first): ....

Double Quiz 4

Question 1

Suppose the probability of it raining today is $0 < p < 1$ if it rained the last two days, $0 < q < 1$ if it rained exactly one out of the last two days, and $r = 1 - q - p$ if it did not rain in the last two days ($r > 0$). Write down a Markov transition matrix P for the relevant Markov chain.

Question 2

Use the transition matrix P from Question 1. Suppose you know it did not rain two days ago, but it rained yesterday. Also, you've been studying in a cubby-hole the last 24 hours, so you do not know whether it rained or not today. What is the probability it will rain tomorrow? Hint: Use the law of total probability.

Question 3

Consider a Markov chain with states $\{1, 2, 3, 4\}$ with transition matrix

$$P = \begin{bmatrix} 1/3 & 1/9 & 2/9 & 1/3 \\ 1/5 & 2/5 & 1/5 & 1/5 \\ 0 & 1/4 & 1/2 & 1/4 \\ 1/8 & 3/8 & 0 & 1/2 \end{bmatrix}$$

Find the probability $P(X_4 = 4, X_3 \geq 3, X_2 \geq 3, X_1 \geq 3 | X_0 = 4)$. Hint: Think about this for a second, before doing any heavy matrix multiplication. It might help to draw the graph of vertices and edges associated to this chain.

Question 4

Show that if an irreducible, aperiodic Markov chain is doubly stochastic with states $\{1, \ldots, M\}$, then $\pi_i = 1/M$ for all i, where the vector π is the stationary distribution. You may assume that a stationary distribution is always unique.

Question 5

Consider a Markov chain with transition matrix

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ .2 & 0 & .8 & 0 \\ 0 & .2 & 0 & .8 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Suppose the states are $\{1, 2, 3, 4\}$ (in the obvious way). (A) If the chain starts in state 2, what is the probably it will visit 4 before visiting 1? (B) What are the transient and recurrent states of this chain?