MA 125 CALCULUS I

Nov 21, 2011

Name (Print last name first): .....

Student Signature: .....

No calculators are allowed!

Part I consists of 7 questions each worth 5 pts. Clearly write your answer (only) in the space provided.

(1) Find all the critical numbers of $f(x) = x^4 - 8x^2$.

Answer: .....

(2) The function $x - x^2$ satisfies the hypothesis of the Mean Value Theorem on $[1, 3]$. Find the number c that satisfies the conclusion of the Mean Value Theorem.

Answer: .....

(3) Find the absolute minimum value of $f(x) = x^3 - 3x + 3$ on interval $[-2, 2]$. (Be sure that your answer is the y-coordinate!)

Answer: .....

(4) Find the open interval on which the function $g(x) = 2x - \ln(x)$ is decreasing. Clearly indicate the endpoints of your interval!

Answer: .....

(5) Find the part of the x-axis on which the function $h(x) = 2xe^{-x}$ is concave up.

Answer: .....

(6) Find the most general antiderivative of $f(x) = 1 + e^{-x} + \cos(x)$.

Answer: .....

(7) Find the most general antiderivative of $f(x) = \frac{1}{1+x^2} - 5x^{-2/3} + e^x$.

Answer: .....

Part II consists of 4 problems. Problem 1 is worth 17 points and problems 2-4 are each worth 16 points. You must show your work on this part to get full credit. Displaying only the final answer without the relevant steps will not get credit.

Problem 1[17pts]

Suppose

$$f(x) = xe^{-x}.$$

(a) Find the x and y intercept(s).

(b) Find all horizontal asymptotes (there are no vertical asymptotes). Hint: use L'Hospital's Rule.

(c) Find the (open) intervals of increase, and (open) intervals of decrease. Hint: factoring the derivative will help.

(d) Find all local maximum and minimum values (both the values and where they occur).

(e) Find the (open) intervals where the function is concave down and the (open) intervals where it is concave up. Hint: factoring will help.

(f) Find the inflection point (both x and y coordinate).

(g) Use (a)-(f) plus the y-intercept to sketch the graph (don't worry about roots).

Problem 2[16pts]

(a) Find the dimensions of a rectangle with perimeter 100ft whose area is as large as possible.

(b) Find two numbers whose product is 81 and whose sum is minimal.

Problem 3[16pts]

Suppose the derivative of $f(x)$ is

$$f'(x) = (x-3)^2(x+2)^3(x-1)^4.$$

(a) Find all the critical numbers of $f(x)$

(b) On what interval(s) is the function $f(x)$ decreasing?

(c) On what intervals(s) is the function $f(x)$ increasing?

Problem 4[16pts]

An object moves along a a straight line with acceleration

$$a(t) = 2\sin(t) + \cos(t).$$

Use antiderivatives to find:

(a) The velocity function $v(t)$ of the object if $v(0) = 0$.

(b) The position function $s(t)$ of the object if its initial position is $s(0) = 0$.