Final Exam

(1) [12pts]True/False

(a) If Q is orthogonal then its determinant is ± 1.

(b) If a 2x2 matrix has only 1 eigenvalue, it is not diagonalizable

(c) If a 3x3 matrix A has an eigenvalue with geometric multiplicity 3 then there exists a basis (for $\mathbb{R}^3$) of eigenvectors of A.

(d) If the span of a set of vectors has dimension n, then there are n vectors in that set

(e) The span of n linearly independent vectors has dimension n

(f) If A has determinant 0, it is not diagonalizable.

(2)[9pts]
Use row reduction to find all (if any) solutions to:

$$x_1 + 2x_2 + 3x_3 = 9$$

$$4x_1 + 5x_2 + 6x_3 = 24$$

$$3x_1 + x_2 - 2x_3 = 4$$

(3)[9pts] Using any method, find the inverse of

$$\begin{pmatrix} 3 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & -1 \end{pmatrix}$$

(4)[9pts] Find the volume of the parallelopiped formed by the three vectors $(1,1,1)$,$(-1,-1,2)$,$(2,0,-1)$ (the tails of the vectors are at the origin).

(5)[8pts] Write a vector equation of the line that contains the points $(1,2,3)$ and $(4,5,4)$ in $\mathbb{R}^3$.

(6)[10pts] Are the following vector spaces or not? Circle those that are, X through those that aren't. (everything is with its usual addition and scalar multiplication)

(a) The plane $x + y + z = 1$

(b) The set of matrices of the form $\begin{pmatrix} a & 2b \\ c & 0 \end{pmatrix}$ $a, b, c \in \mathbb{R}$

(c) The subset of matrices in $V = M_{nn}$ which are upper triangular

(d) The set of continuous functions on $[0, 1]$ such that $f(1) = 0$

(e) The set of vectors in the span of $\{v_1, v_2\}$

(7)[8pts] Write down a definition of **linear independence** for a set of vectors.

(8)[15pts] Consider the bases $B_1 = \{(3, 1), (2, -1)\}$ and $B_2 = \{(2, 4), (-5, 3)\}$.
(a) If a vector v in the first basis is $(v)_{B_1} = (a, b)$, then what is $(v)_{B_2}$?

(b) What is the change of basis matrix (from B_1 to B_2)

(9)[20pts] Find the characteristic polynomial of A. Find a diagonal matrix D and an orthogonal matrix C such that $A = C^{-1}DC$. (Hint: C is a rotation matrix)

$$A = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}$$