(1)[15pts] Write down 3 conditions which are equivalent to a square matrix being invertible.

(2)[15pts] Given $D = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix}$, write down the matrices A, B, C such that

$$AD = \begin{pmatrix} a & b & c \\ d & e & f \\ g+2a & h+2b & j+2c \end{pmatrix}$$

$$BD = \begin{pmatrix} g & h & j \\ d & e & f \\ a & b & c \end{pmatrix}$$

$$CD = \begin{pmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3j \end{pmatrix}$$

(3)[15pts] If $v = (1, 2, 1)$ and $u = (2, 3, 1)$, calculate the projection vector $\text{proj}_v u$.

(4)[2pts each] Circle the vector spaces, X those that aren't (no explanations needed):

(a) $\{(0, 0, 0)\}$

(b) the number 1

(c) the polynomials $\{ax^2 + bx + c : a^2 = b\}$

(d) the continuous functions $\{f \in C([0, 1]) : f(0) = f(1)\}$

(e) the set of vectors in $\mathbb{R}^3$ with z-coordinate equal to 1

(f) the set of vectors (a, b, c) in $\mathbb{R}^3$ satisfying $a + b + c = 0$

(g) the set of 5×5 matrices which have 0's along the diagonal, i.e. $a_{ii} = 0$

(h) the set of 5×5 matrices whose sum along the diagonal is 0, i.e. matrices satisfying

$$\sum_{i=1}^{5} a_{ii} = 0.$$

(5)[15pts] Write an equation for the span of $\{(1,2,1),(-1,-4,0)\}$.

(6)[24pts] Recall $\mathbb{P}_2 = \{ax^2 + bx + c : a, b, c \in \mathbb{R}\}$. Prove that the set of polynomials

$$\mathbb{P}_2 = \{ax^2 + bx + c : 3a - b + 2c = 0\}$$

is a subspace of $\mathbb{P}$.

(Bonus)[+10pts] Use a 2x2 determinant to calculate the area of the triangle formed by the three vertices $(1,2),(3,5),(0,7)$. Hint: the diagonal of a parallelogram splits it into 2 congruent triangles.