

## Quiz 5 and 6

(1) [14pts] True/False

(a) If  $Q$  is symmetric then  $Q$  is orthogonal.

(b) If  $Q$  is orthogonal then its determinant must be 1.

(c) If a  $3 \times 3$  matrix  $A$  has an eigenvalue with geometric multiplicity 3 then there exists a basis (for  $\mathbb{R}^3$ ) of eigenvectors of  $A$ .

(d) An eigenvalue  $\lambda$  can never be equal to 0.

(e) If  $\det A = 0$  then 0 is an eigenvalue of  $A$ .

(f) If a  $3 \times 3$  matrix has only 2 distinct eigenvalues, it is not diagonalizable.

(g) If  $A$  has determinant 0, it is not diagonalizable.

(2)[5pts] Write down a definition of an **orthogonal matrix**.

(3)[15pts] Find the characteristic polynomial of the matrix below. Also write down the algebraic and geometric multiplicities of the eigenvalue(s).

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

(4)[6pts] Given that  $A$  is diagonalizable. Write down the diagonal matrix  $D$  which it is similar to.

$$A = \begin{pmatrix} 1 & 2 \\ 0 & c \end{pmatrix}$$