HOMEWORK 1
FINANCIAL MATHEMATICS MA 492
DUE OCT. 1

Solutions are graded on WORK/METHOD/REASONING and not on your final numerical answer, so please justify your steps! Answers to book problems are in the back of the book. You may refer to these BEFORE your write-up, but not DURING. The same applies to any help from classmates or the internet. Remember, 1/2 the exam problems (for MA 492) will be very similar to book problems, and you will NOT be able to use the book during exams.

Book problems:

Ch. 1: 2, 5, 7a, 14

Ch. 2: 8, 12, 21, 22, 23, 24, 28, 30, 31, 32

Ch. 3: 3, 5, 8, 10, 11, 12, 13

In "The Mathematics of Gambling," Thorp considers a strategy for blackjack where the probability that the player wins any given bet is $p = 50.05\%$.

Assuming this, if you have \$100 to gamble, and on one turn you gamble $100x$ dollars, where x is a number in $[0, 1]$, then your resulting fortune after one bet is:

$$X = \begin{cases} 100(1+x) & \text{with probability } 0.5005, \\ 100(1-x) & \text{with probability } 0.4995. \end{cases}$$

a) Find the expectation $\mathbf{E}X$ as a function of x.

b) What value of x in $[0, 1]$ maximizes the expectation of X?

c) Let us define the random variable

$$k = \log(X/100),$$

which is called the "logarithmic return rate." Calculate $\mathbf{E}k$ as a function of x.

d) Find the value of $x \in [0, 1]$ to maximize $\mathbf{E}k$.

Extra problems for MA 592 students:

- For random variables that can take at most n values, show that $\mathbf{E}(X+Y) = \mathbf{E}(X)+\mathbf{E}(Y)$. If μ_X is the mean of X, show that $\mathbf{E}(X-\mu_X)^2 = \mathbf{E}(X^2)-\mu_X^2$.
- Suppose X takes at most countably many values. Show that $\mathbf{E}(X^2) < \infty$ implies $\mathbf{E}(X) < \infty$.
- If A is an $n \times n$ matrix and $b \in \mathbb{R}^n$, show that $Ax = b$ has a unique solution in $\mathbb{R}^n$ if and only if $\det A \neq 0$.
- Prove L'Hôspital's Rule.
- Prove that a monotone increasing sequence always has a (possibly infinite) limit.