HOMEWORK 2 MA 492
DUE OCT. 17

Solutions are graded on WORK/METHOD/REASONING and not on your final numerical answer, so please justify your steps! Answers to book problems are in the back of the book. You may refer to these BEFORE your write-up, but not DURING. The same applies to any help from classmates or the internet. Remember, 1/2 the exam problems (for MA 492) will be very similar to book problems, and you will NOT be able to use the book during exams.

Book problems:

Ch. 3: 15
Ch. 5: 1-4, 10, 12, 13, 16, 17

Three stocks have the same starting price $S_1(0) = S_2(0) = S_3(0) = 100$. You invest $V(0) = 100$ in a portfolio consisting of fixed quantities of these 3 stocks:

$$V(t) = x_1 S_1(t) + x_2 S_2(t) + x_3 S_3(t)\,.$$

So, defining

$$\boldsymbol{u} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

you have constraint 1: $\boldsymbol{u}^T \boldsymbol{x} = 1$.

Suppose the stocks have expected returns

$$\boldsymbol{v} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.3 \end{bmatrix}$$

Let constraint 2 be: $\boldsymbol{v}^T \boldsymbol{x} = 0.2$.
(your desired expected return is 0.2.)

Also suppose that the covariance matrix for the returns is

$$C = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 4 & 2 \\ 1 & 2 & 4 \end{bmatrix}$$

which has inverse

$$C^{-1} = \frac{1}{32} \begin{bmatrix} 12 & -2 & -2 \\ -2 & 11 & -5 \\ -2 & -5 & 11 \end{bmatrix}$$

Find vector $\boldsymbol{x}$ which minimizes the variance $\boldsymbol{x}^T C \boldsymbol{x}$ subject to the constraints.

Extra problems for MA 592 students:

Ch. 5: 11, 14, 15

- Prove part 2) of Corollary 3.20 in the text.
- Show that the correlation coefficient satisfies $|\rho_{12}| \leq 1$ (see pg. 62). Hint: think Cauchy-Schwarz inequality.
- Let $\vec{X} = (X_1, X_2)$ be a two-dimensional random vector.

  (a) Covariance is non-negative definite: show that for any non-zero $v \in \mathbb{R}^2$,
  $$v\mathrm{Cov}(\vec{X})v^T \geq 0.$$
  (b) Show that $\mathrm{Cov}(\vec{X})$ has two eigenvalues, both of which are non-negative.

  (c) If A is a 2x2 matrix and $b \in \mathbb{R}^2$ then
  $$\mathrm{Cov}(A\vec{X} + b) = A\mathrm{Cov}(\vec{X})A^T.$$