HOMEWORK 4 MA 492
DUE FRIDAY NOV. 22 5PM
(IN MY MAILBOX AT CH 452)

Solutions are graded on WORK/METHOD/REASONING and not on your final numerical answer, so please justify your steps! Answers to book problems are in the back of the book. You may refer to these BEFORE your write-up, but not DURING. The same applies to any help from classmates or the internet. Remember, 1/2 the exam problems (for MA 492) will be very similar to book problems, and you will NOT be able to use the book during exams.

Book problems:

Ch. 8: 1,2,5,6.

(1) Calculate

$$A = \int_{-\infty}^{\infty} e^{-x^2/2} dx.$$

Hint: Note that this is equal to $\int_{-\infty}^{\infty} e^{-y^2/2} dy$, so multiplying the two together gives A^2. Next, use Fubini's theorem and switch to polar coordinates.

(2) Use integration by parts to calculate

$$\int_{-\infty}^{\infty} x^2 e^{-x^2/2} dx.$$

(3a) Suppose the mean on an exam is 60. Use Markov's inequality to get a bound on the probability that a given test score is greater or equal to 85.

(3b) Use Chebychev's inequality to get a lower bound on the probability that a given test score is between 51 and 69

(4) A die is rolled 50 times. Use the Central Limit Theorem (with the histogram correction) to estimate the probability that the sum is (a) strictly less than 180, (b) strictly less than 195.

(5) Calculate the moment generating function (MGF) for a standard normal random variable $N(0, 1)$.

(6) [592 required, 492 can do this for extra credit] Problem 2 above is the second moment. Now, find a formula for the general nth moment of a standard normal. Hint: this is easier to do directly than with the MGF. Set up the integral and than use the u-substitution $u = x^2/2$. You may then use known results about the Gamma function (look this up on Wikipedia if you don't know what this is).